Open Problems about the Simplex Method

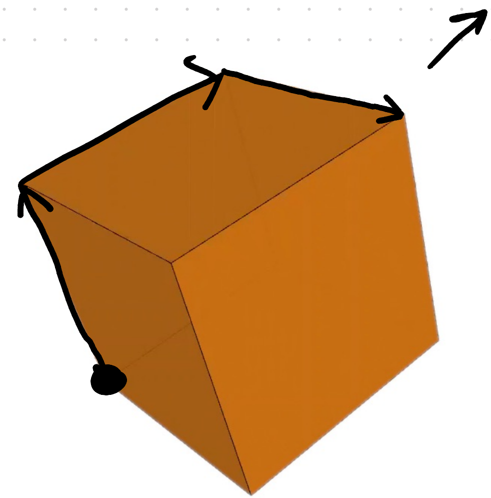
Sophie Huiberts
CNRS, LIMOS

Linear programming

maximize $c^T x$
subject to $Ax \leq b$

we get $A \in \mathbb{R}^{n \times d}$
$b \in \mathbb{R}^n$
$c \in \mathbb{R}^d$

we compute $x \in \mathbb{R}^d$



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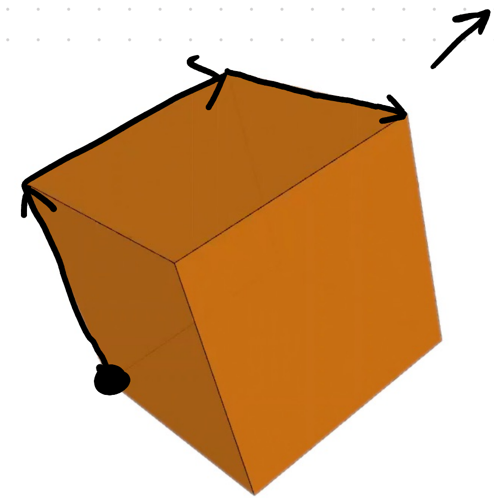

how many pivot steps?

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ratio test

In practice

The simplex method takes $2(n+d)$ pivot steps to solve an LP.

Worst-case complexity

Theorem The simplex method has exponential worst-case complexity

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*terms and conditions apply

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Maximum subdeterminants

Does theory reflect reality?

0. on true but useless theorems

1. an instancewise assumption

2. a distributional assumption

3. "real" simplex method

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To start:

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A: Under reasonable assumptions, at most $1386 \cdot 10^6 \ km^3$

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Suppose V is the vertex set of feasible region

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Optimal solution after $\frac{\Delta}{\delta} d^2 \log(d \frac{\Delta}{\delta})$ steps

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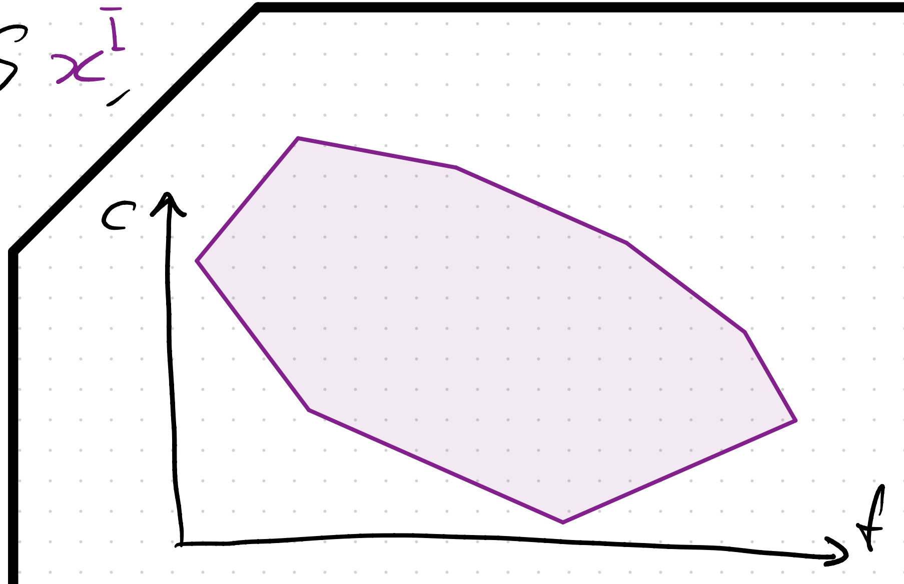

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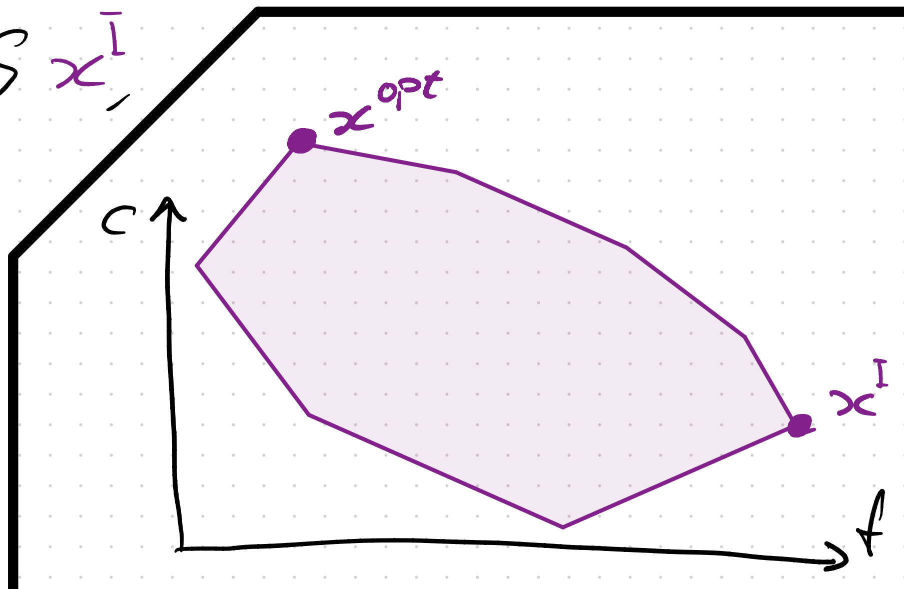

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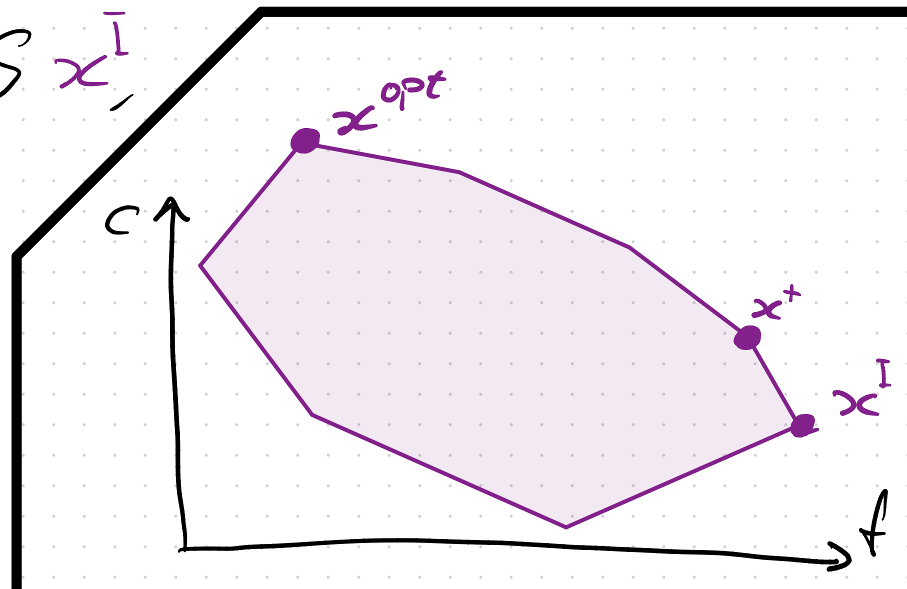

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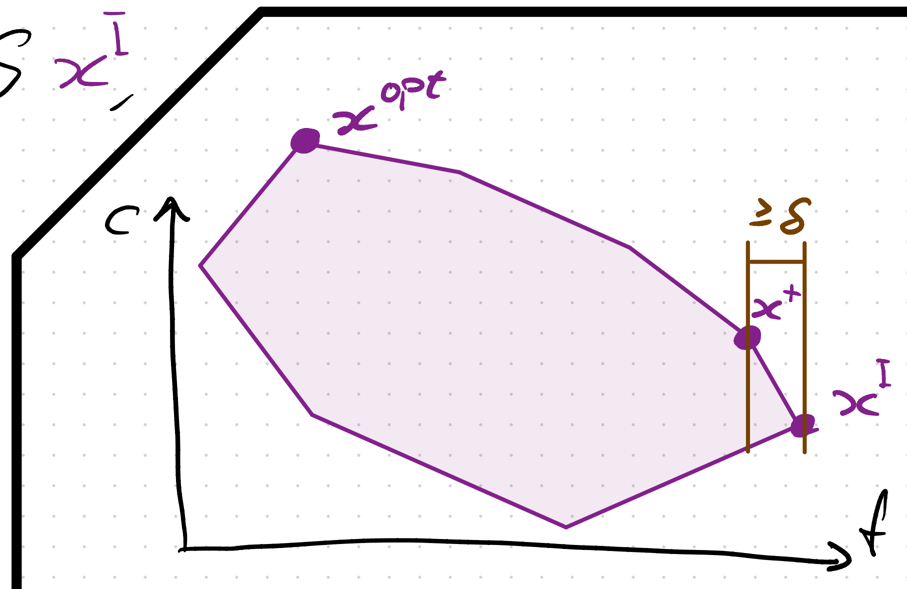

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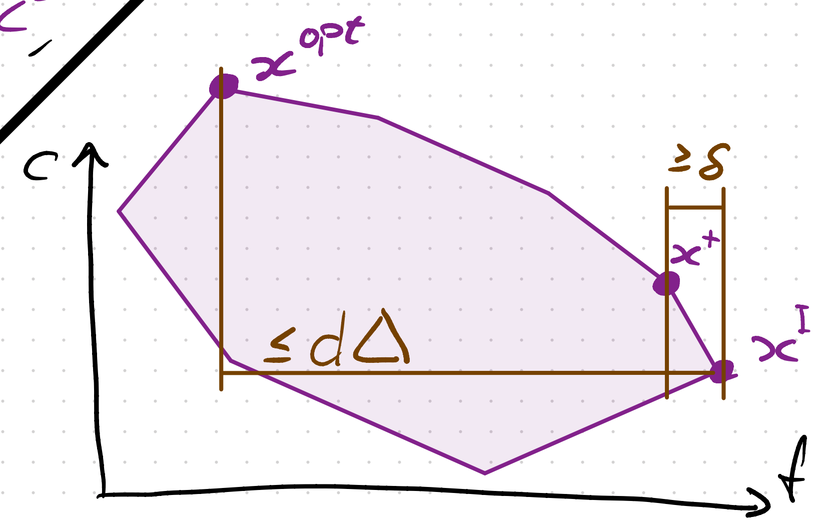

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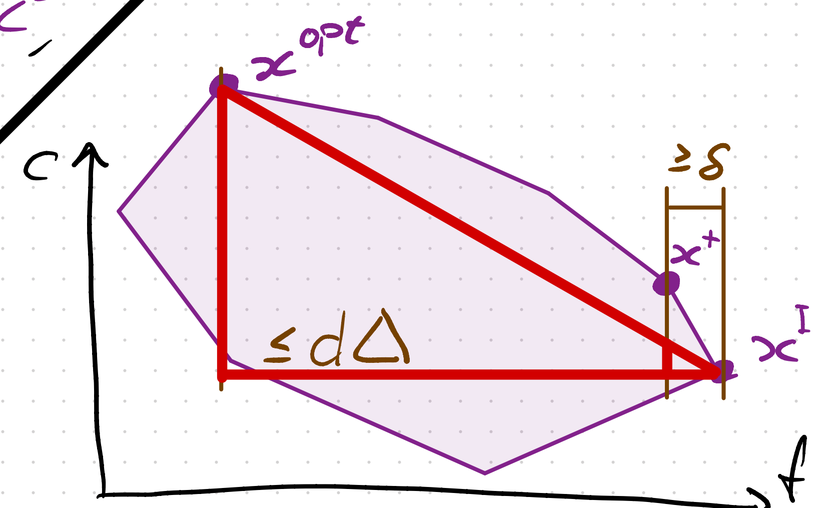

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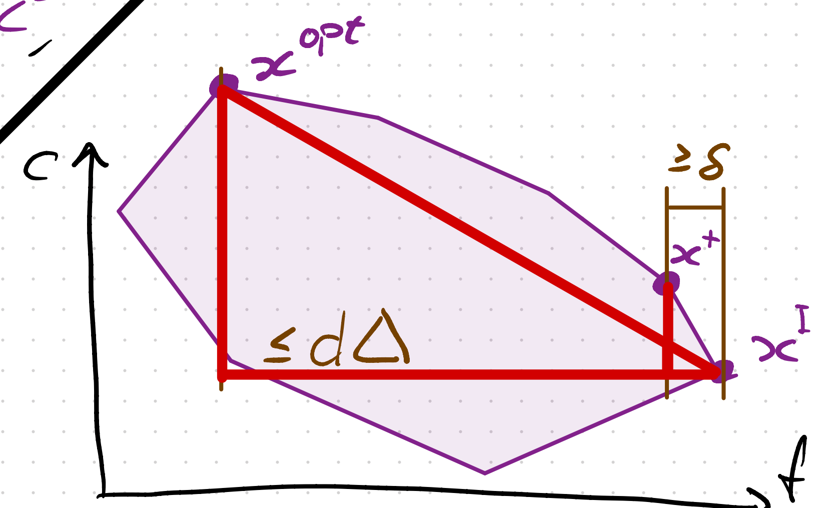

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Kuno, Sano & Tsuruda proved computing δ is NP-Hard

A posteriori δ

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afiro	27×59	16
kb2	43×84	61
sc50b	50×98	49
blend	74×157	92
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scagr7	129×269	140
sc205	205×408	236
beaconfd	173×435	102
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Kuno
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CAN conclude bound is far from tight

CAN NOT conclude much else

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Performance variability

Sometimes the algorithm gets lucky and is faster than usual

Sometimes it gets unlucky

Open Questions:

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Are these the right questions to ask?

Does theory reflect reality?

0. on true but useless theorems

1. an instancewise assumption

2. a distributional assumption

3. "real" simplex method

Smoothed analysis

Let $\bar{A} \in \mathbb{R}^{n \times d}$ have rows of norm ≤ 1.

$\bar{b} \in [-1, 1]^n$, $c \in \mathbb{R}^d$

Let $\hat{A}$, $\hat{b}$ have i.i.d $N(0, \sigma^2)$ entries.

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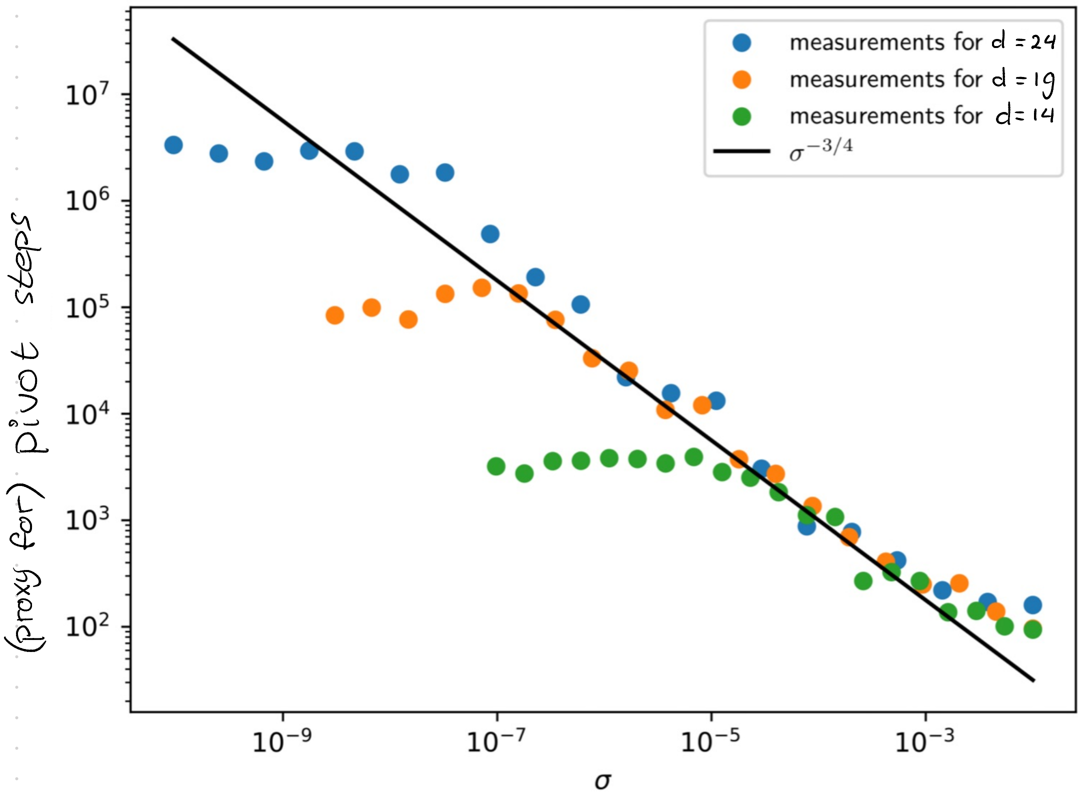
much effort spent to get stronger bounds

Can any of the resulting insights be tested experimentally?

Synthetic data



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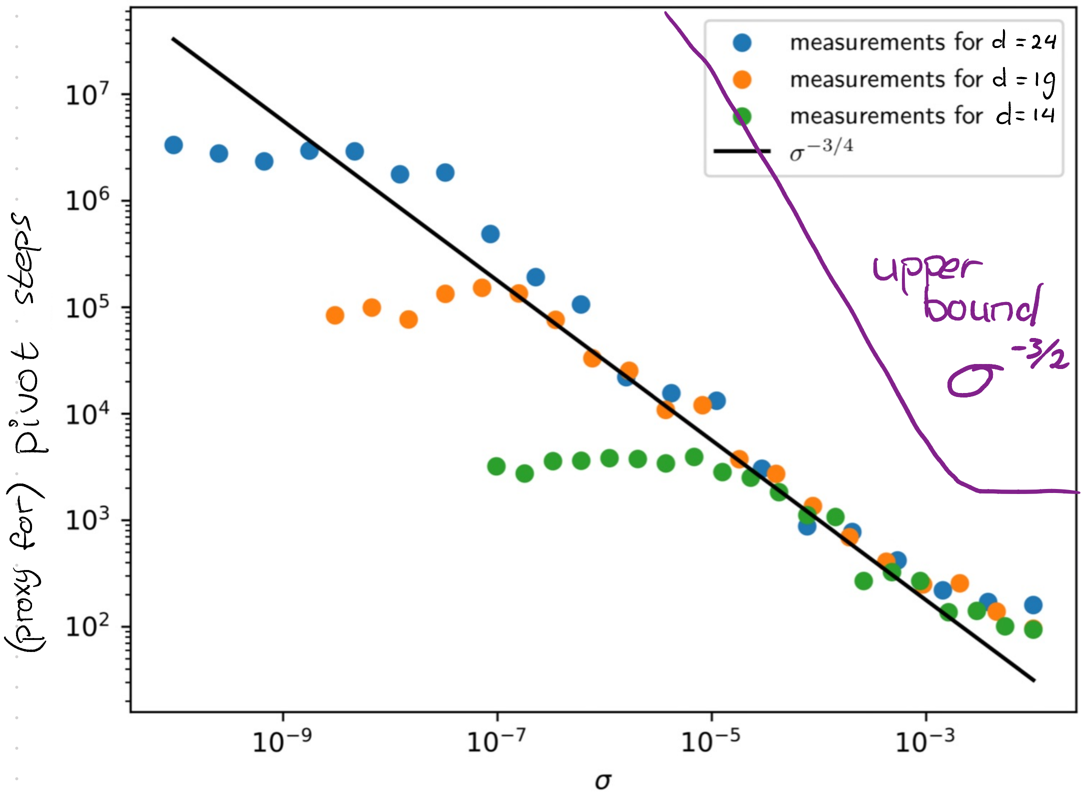

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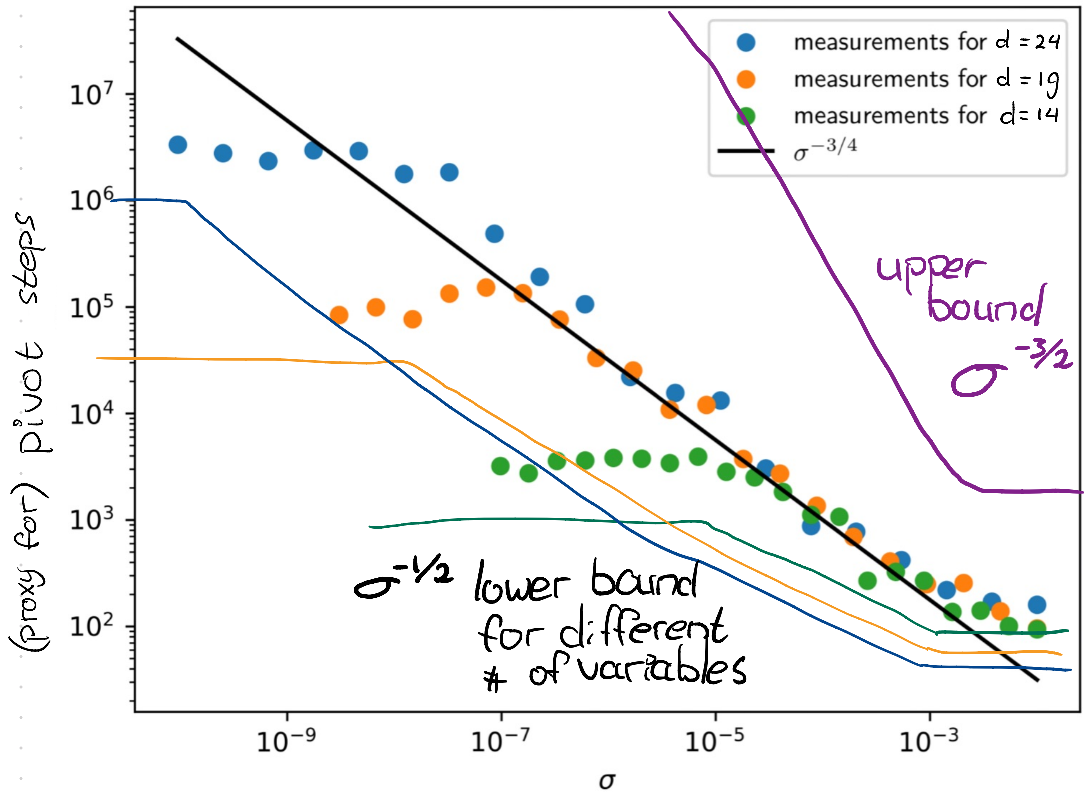

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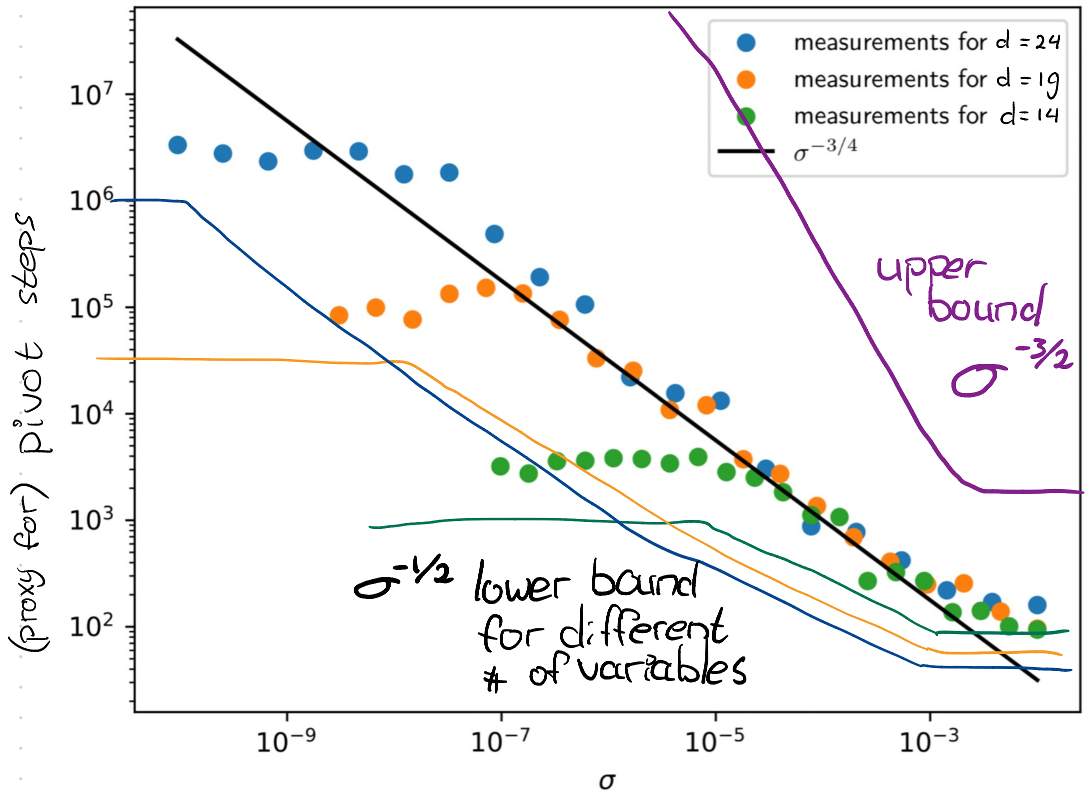

CAN tell if theorem is tight

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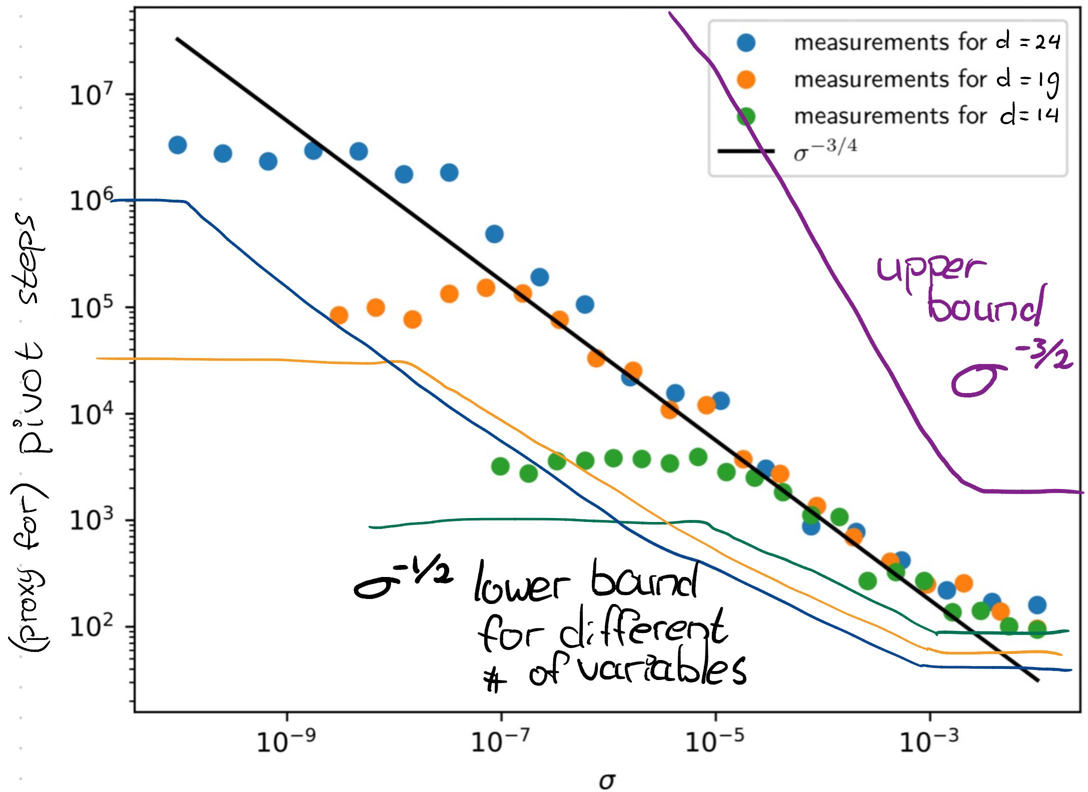

CAN tell if theorem is tight

CAN NOT tell if theorem is useful

Do there exist

Gaussian distributed

linear programs

in real life?

The first linear program

Given 77 ingredients,
find the cheapest diet
that meets all 9 nutrient needs

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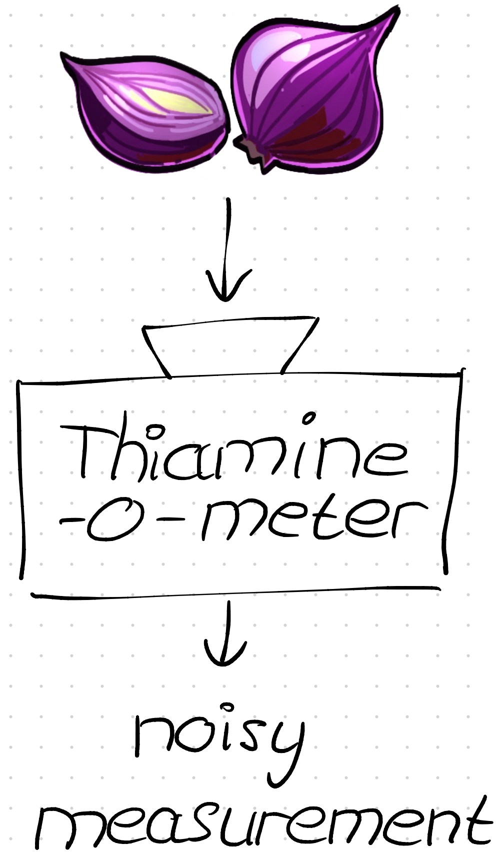

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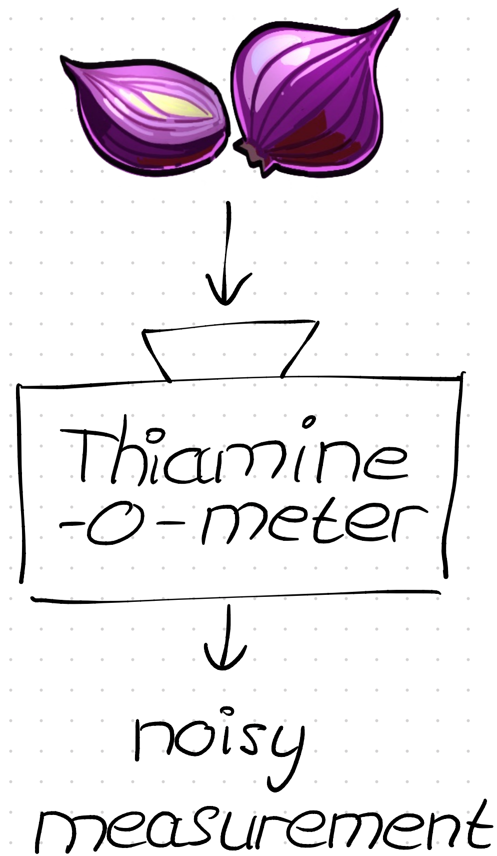

why would anyone
do this?

George Stigler (1911 - 1991)

THE quintessential conservative

opposed to rent-control
& price controls

ON THINKING ABOUT GEORGE STIGLER*

C. R. McCann, Jr. and Mark Perlman

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ON THINKING ABOUT GEORGE STIGLER*

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opposed to minimum wage

THE ECONOMICS OF MINIMUM WAGE LEGISLATION

By George J. Stigler*

THE ECONOMICS OF MINIMUM WAGE LEGISLATION

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One final point: We seek to abolish poverty in good part because it leads to undernourishment. In this connection, dietary appraisals show that in any income class, no matter how low, a portion of the families secure adequate diets, and in any income class, as high as the studies go, a

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[26] Tax-supported bureaucrats and professors may also have another reason for certain of their practices.

The first linear program

Given 77 ingredients,
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↓

Thiamine-o-meter

↓

noisy measurement

Who else did this?

SLIDE 1

Ministry of Defense
Coordination of Government Activities in the Territories

Food Consumption in the Gaza Strip – Red Lines

1 January 2008

SLIDE 2

Goals of Analysis

- **As part of the policy formulated by the Security Cabinet on September 19, 2007, Israel will limit the entry of goods into the Gaza Strip.**
- **In order to allow for a basic fabric of life in the Gaza Strip, the deputy defense minister approved allowing 106 trucks carrying basic humanitarian products into the Gaza Strip, mostly food (all products are specified in the appendices). In addition, food in seed form was approved for entry via the aggregate conveyor belt located near the Karni crossing.**
- **This research examines the main food component.**
- **The goal of the analysis – to identify the point of intervention for prevention of malnutrition in the Gaza Strip.**
- **The basis for the analysis is a model formulated by the Ministry of Health (at this point, according to average Israeli consumption) and a model formulated by the Palestinian Ministry of Economy.**
- **The Ministry of Health is conducting work for calculating the <u>minimal</u> subsistence basket based on the Arab sector in Israel. <u>The "minimum basket" allows nutrition that is sufficient for subsistence without the development of malnutrition</u>.**

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List of humanitarian products whose transfer into the Gaza Strip is permitted - May 30, 2010

Basic humanitarian food products	Comments
Flour, semolina, wheat and yeast	
Oil, pasta, rice, salt, sugar and saccharine	
Frozen meat and chicken products and fish	
Sausage, canned meat and fish	
Milk products, margarine, milk powder and baby food	
Legumes: broad beans, soy (grains, powder) garbanzo beans, lentils, peas, beans and lupine	
Grains: barley, corn (kernels/ground), oatmeal, sorghum	
Fruit: apples, pears, bananas, loquat, moist dates and avocado, apricots, plums, green almonds, kiwi, mango, pomegranate	Kerem Shalom has a daily quota of 22 agricultural trucks including fruit, vegetables and agricultural inputs
Fresh vegetables: carrots, garlic, pumpkin and onion, green leaves (coriander, dill, parsley, etc.)	Kerem Shalom has a daily quota of 22 agricultural trucks including fruit, vegetables and agricultural inputs
Frozen vegetables	
Processed garbanzo beans without additives or tahina	
Tea and coffee	
Halva, jam	
Basic canned goods	
Eggs for consumption	
Containers and bottles of mineral water	
Spices: black pepper, soup powder, za’atar, sesame, cinnamon, anise, chamomile, sage	
Permitted for donations and international organizations only	
Vitamin enriched biscuits and bottled water	
Tomato paste	

translation by Gisha

SLIDE 6

General Daily Food Consumption in the Gaza Strip per Ministry of Health Scale (in tons)

Age/Type of food	Male/Female			Female			Male					Total for general population (minus 6-12 month age bracket)	Food additive for -12 month age bracket)	Total quantity required for general population
	2-3	4-6	7-10	11-24	24-50	51+	11-14	15-18	19-24	24-50	51+			
Grains	11.94	37.15	40.43	63.94	53.52	14.65	25.66	25.71	25.71	68.33	15.23	382.28	3.98	386.26
Vegetables	12.62	37.00	40.52	60.03	50.25	14.64	24.64	24.68	24.68	65.61	14.85	369.53	4.21	373.74
Fruit	16.99	58.80	67.42	102.65	85.92	23.84	43.69	45.02	45.02	119.68	25.80	636.86	6.33	643.19
Milk	39.49	70.18	68.53	140.88	78.61	26.18	51.40	26.34	46.34	82.13	22.13	672.22	13.16	685.38
Meat	14.09	39.83	62.61	60.93	51.00	15.57	24.08	23.38	23.38	62.15	14.95	371.98	4.70	376.67
Oil	0.00	1.18	1.81	0.00	3.39	0.56	0.82	2.22	1.85	7.87	1.06	20.75	0.00	20.75
Sugar	4.35	5.04	5.58	4.95	5.27	2.01	2.87	5.18	4.07	12.78	1.85	53.95	1.45	55.40

- **The figures are in tons per calendar day (consumption over seven days per week, unlike supply which is calculated based on five days per week).**
- **The portion of consumption is measured by the Health Ministry in Israel and provides for 2,000-2,500 calories per adult and 1,550 calories per child.**
- **The quantities in this table are average consumption according to Israeli standards and are not minimal subsistence portions.**
- **The Ministry of Health has been requested to calculate the minimal subsistence basket according to the Arab sector in Israel. The "minimal basket" allows for nutrition that is sufficient for subsistence without the development of malnutrition.**

SLIDE 13

Additives in Wheat

Number	Added Vitamin/Mineral	Quantity	
1	Thiamine (Vitamin B1)	4.4	Milligram per Kilogram
2	Vitamin B2	2.6	Milligram per Kilogram
3	Niacin	35	Milligram per Kilogram
4	Folic Acid	0.4	Milligram per Kilogram
5	Iron	25	Milligram per Kilogram
6	Folato	1	Milligram per Kilogram
7	Vitamin B6	2.5	Milligram per Kilogram
8	Zinc	15	Milligram per Kilogram
9	Vitamin A	1	Milligram per Kilogram
10	Vitamin B3	0.02	Milligram per Kilogram

SLIDE 14

Summary and Conclusions

- **According to the model supplied by the Israeli Ministry of Health, there is a need for a daily supply of 104 food trucks (5 days a week).**
- **The model takes into account an exaggerated consumption of milk (3 times the known consumption in the Gaza Strip). Thus, on decreasing the milk component, the working assumption of 106 trucks (+ Karni conveyor belt) which includes about 90 truckloads of basic food, certainly meets nutritional needs in the Gaza Strip.**
- **The Ministry of Health Model assumes lower consumption of flour than what is known to be in effect.**
- **The Ministry of Health model is based on the average Israeli consumption, rather than a minimalist basket according to consumption habits in the Arab sector (the Ministry of Health is currently analyzing this).**
- **Following receipt of the new basket, it will be possible to define a red line as a warning sign.**
- **The Ministry of Health estimates that the new basket will be 20% lower than the current basket.**

“We Didn’t Want to Hear the Word ‘Calories’”: Rethinking Food Security, Food Power, and Food Sovereignty—Lessons from the Gaza Closure*

Aeyal Gross** and Tamar Feldman***

גישה
مسلك gisha

Gisha – Legal Center for Freedom of Movement
גישה - מרכז לשמירה על הזכות לנוע (ע״ר)
مسلك - مركز للدفاع عن حرية الحركة

ON PAPER | ورقة موقف | נייר עמדה

Reader: "Food Consumption in the Gaza Strip - Red Lines"

October 2012

“Humanitarian Minimum”

Israel’s Role in Creating Food and Water Insecurity in the Gaza Strip

December 2010

רופאים לזכויות אדם - ישראל (ע״ר)
أطباء لحقوق الانسان-إسرائيل
Physicians For Human Rights - Israel

The first linear program

Given 77 ingredients,
find the cheapest diet
that meets all 9 nutrient needs

Thiamine-o-meter

noisy measurement

Is this legal?

INTERNATIONAL COVENANT ON ECONOMIC, SOCIAL AND CULTURAL RIGHTS

1967 human rights treaty

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OFFICE OF THE HIGH COMMISSIONER FOR HUMAN RIGHTS

CESCR General Comment No. 12: The Right to Adequate Food (Art. 11)

1999 explanation of these rights

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1967 human rights treaty

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1999 explanation of these rights

The *right to adequate food* shall therefore not be interpreted in a narrow or restrictive sense which equates it with a minimum package of calories, proteins and other specific nutrients.

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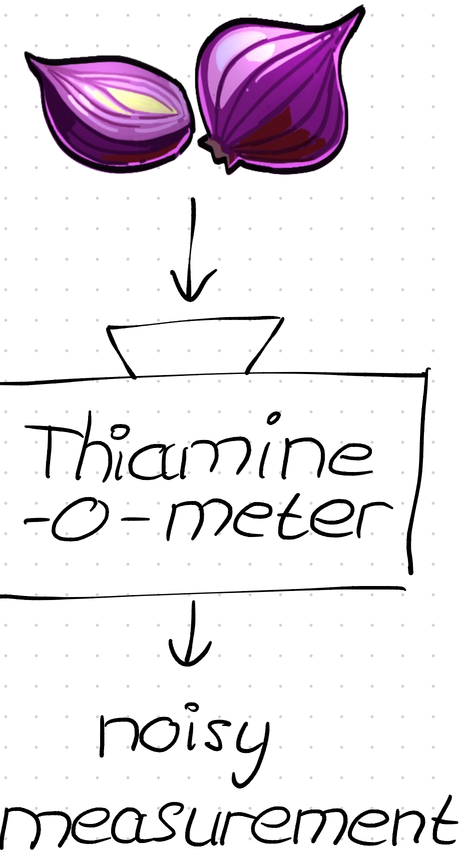
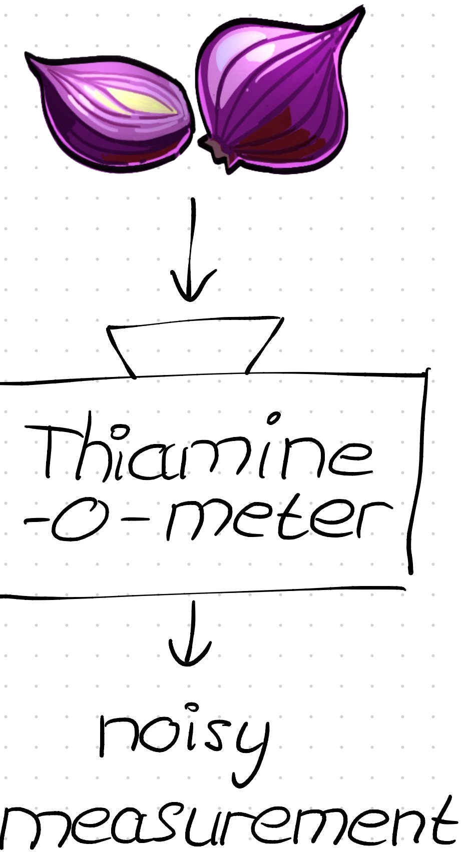
First simplex method

Type of Operations	No. of repetitions
Multiplication	15,315
Division	1,234
Addition of two numbers	14,561
Addition of 77 numbers	190
Addition of 9 numbers	85

Math Tables Project, 1948



Could smoothed analysis have predicted that the first computation be fast?



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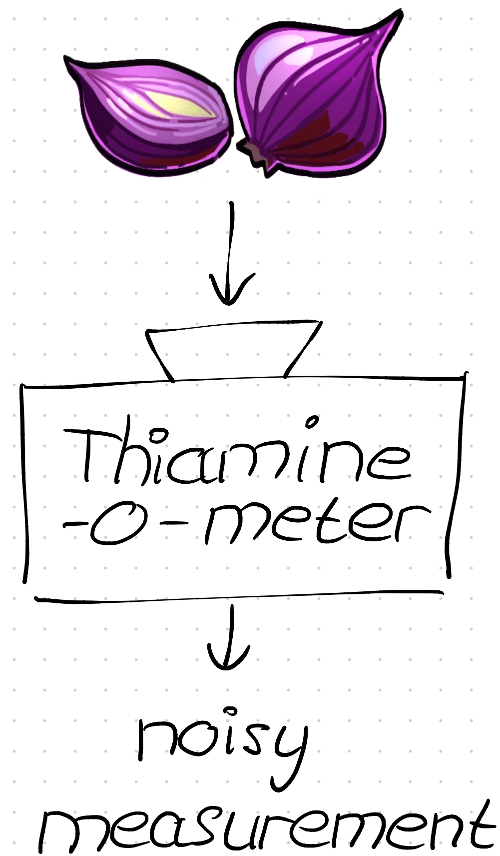

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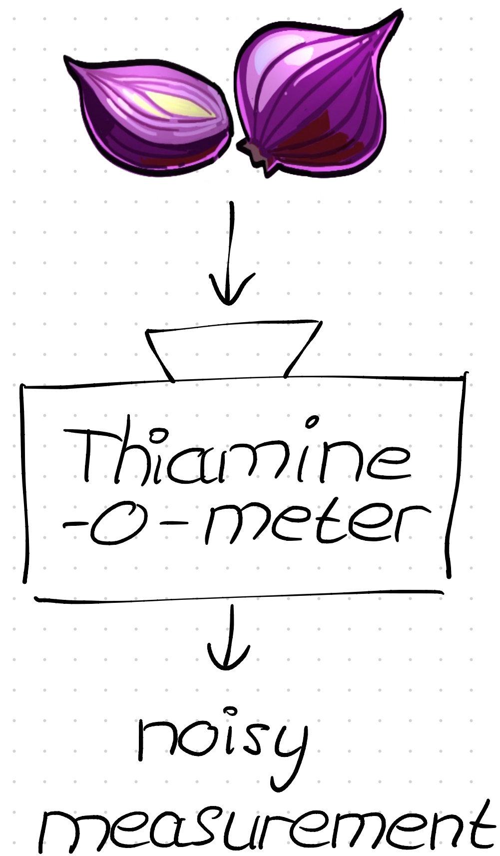

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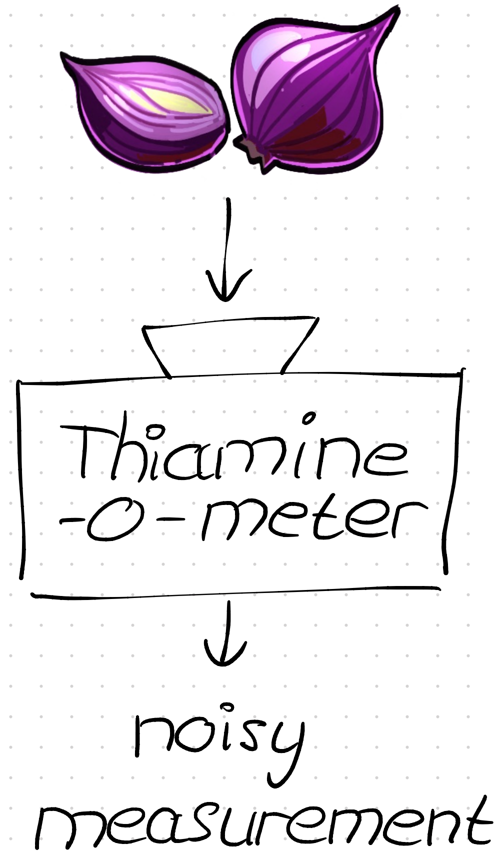
2a. non-negativity constraints



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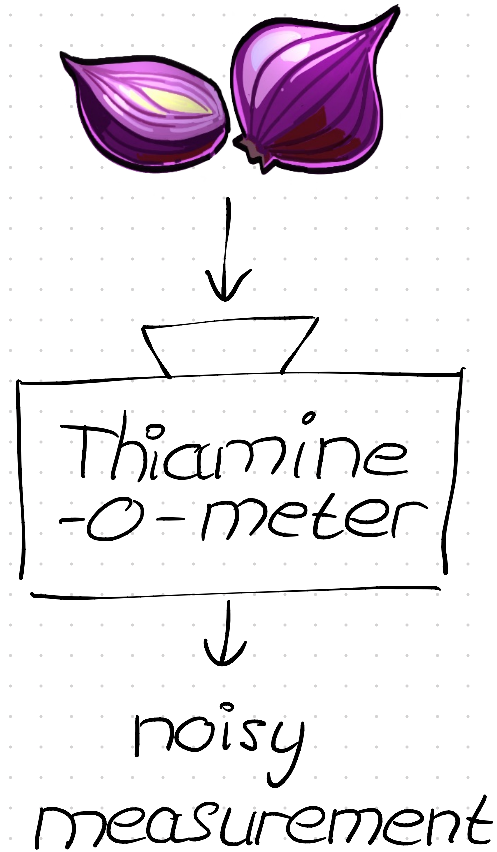
2b. multiplicative error $\geq 15\%$, but need additive error



Open Questions

What would it mean to test smoothed analysis' conclusions?

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What would it mean to test smoothed analysis' conclusions?

Is more noise really better?

Does theory reflect reality?

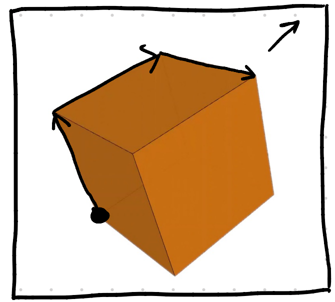
0. on true but useless theorems

1. an instancewise assumption

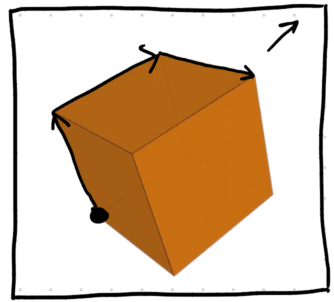
2. a distributional assumption

3. "real" simplex method

Are we studying the simplex method?

This geometry  doesn't exist.

Are we studying the simplex method?

This geometry  doesn't exist.

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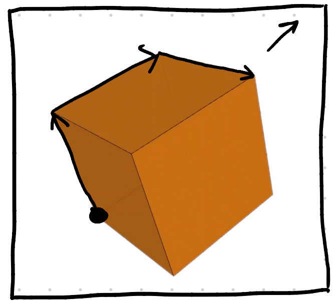
This geometry  doesn't exist.

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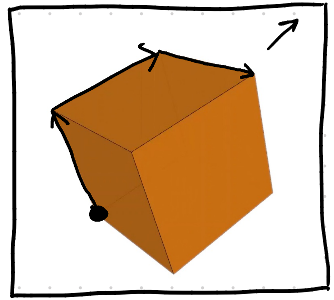
This geometry  doesn't exist.

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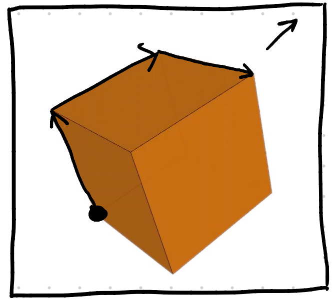
This geometry  doesn't exist.

What does exist:
- numerical linear algebra
- bound shifting
- bound perturbations
- Harris' ratio test

Scaling

The user/software scales the variables & constraints:

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- All variables & slacks have value $0 \leq \cdot \lesssim 1$

Scaling

The user/software scales the variables & constraints:

- All variables & slacks have value $0 \leq \cdot \lesssim 1$
- All colums & rows have $\|\cdot\|_\infty \approx 1$
- ~

What is a basis?

if $\det B > \varepsilon \Rightarrow$ basis ✓

What is a basis?

if $\det B > \varepsilon \Rightarrow$ basis ✓

if $\det B \leq \varepsilon \Rightarrow$ not a basis ✗

Feasibility

if $Ax \leq b$, feasible ✓

Feasibility

if $Ax \leq b$, feasible ✓

if $A_i x > b + 10^{-6}$, infeasible ✗

Feasibility

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otherwise, solver <u>chooses</u>

Optimality

if $A_I^{-T} c \geq 0$, optimal ✓

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otherwise, solver chooses

Accumulating error

Solver maintains $\begin{cases} \text{a sparse LU decomp} \\ \text{solution } x \\ \text{reduced costs} \end{cases}$

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Rank-1 update every pivot step

When errors grows too large, recompute

Bound shifting

What if your current solution $x = A_I^{-1} b_I$ is infeasible?

Bound shifting

What if your current solution $x^I = A_I^{-1} b_I$ is infeasible?

Easy: set $b_i \leftarrow \max(b_i, A_i x^I)$ and proceed as normal

We solved $\max\ c^T x$ for $\hat{b} \neq b$.

st $Ax \leq \hat{b}$,

We solved $\max c^T x$ st $Ax \le \hat{b}$, for $\hat{b} \ne b$.

$\boxed{\text{Most often, basis is already feasible for original LP}}$

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Duality gives $y \in \mathbb{R}^n$, with $y \geq 0$ and $y^T A = c$

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Duality gives $y \in \mathbb{R}^n$,

with $y \ge 0$ and $y^T A = c$

$$\boxed{\text{Feasible for original dual LP!}}$$

Nothing is sacred

If you mess with your bounds, the dual simplex method will solve your problem with a handfull of steps.

Nothing is sacred

If you mess with your bounds, the dual simplex method will solve your problem with a handfull of steps.

How can we use this?

Bound perturbations

Before you start,

set $b_i \leftarrow b_i + \text{Unif}([0, 10^{-6}])$

Bound perturbations

Before you start,
set $b_i \leftarrow b_i + \text{Unif}([0, 10^{-6}])$

This removes all degeneracy

Harris' ratio test

From $I \subseteq [n]$, $|I| = d$, we set $x^I = A_I^{-1} b_I$

I gift you some I^0

for $k = 0, \dots, T$,

  you drop some $i \in I^k$

  you pick some $j \notin I^k$

  you set $I^{k+1} \leftarrow I^k \cup \{j\} - \{i\}$

  you want $Ax^{I^{k+1}} \leq b$

Harris' ratio test

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Paula Harris

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Paula Harris

Harris' ratio test

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I gift you some I^0

for $k=0,\dots,T$,

you drop some $i \in I^k$

maximize determinant

you pick some $j \notin I^k$

you set $I^{k+1} \leftarrow I^k \cup \{j\} - \{i\}$

you want $Ax^{I^{k+1}} \le b + 10^{-6}$

Paula Harris

Open Questions

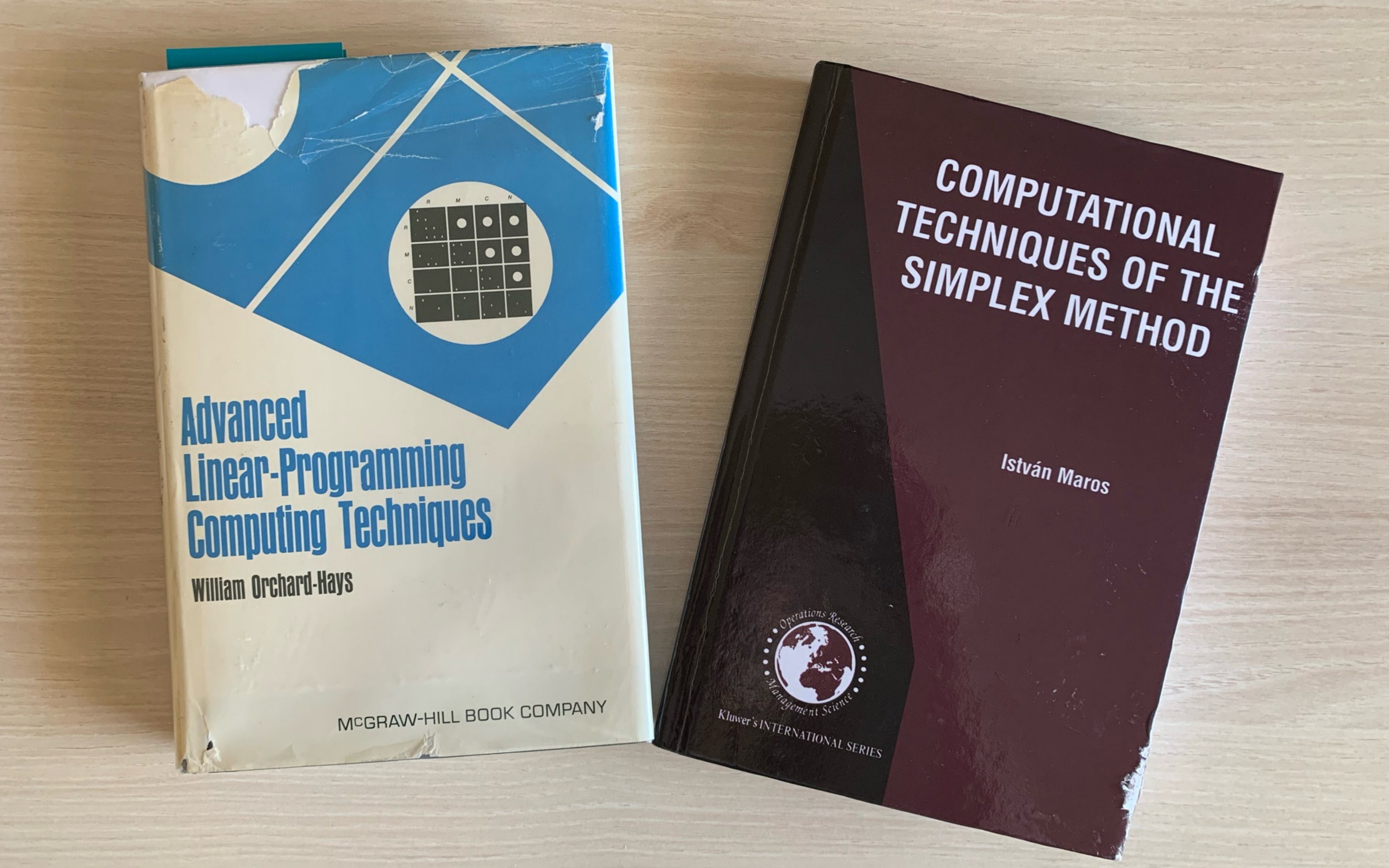
- Why does this terminate in finite time?

Open Questions

- Why does this terminate in finite time?
- Why do these tricks help?

Open Questions

- Why does this terminate in finite time?
- Why do these tricks help?
- Do they help for equal or opposite reasons?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Liverpool
Manchester
Leeds
Birmingham
Cardiff
London
Brighton
English Channel
North Sea
Lindesnes
Gothenburg
Aalborg
Kalmar
Baltic Sea
Copenhagen
Hamburg
Amsterdam
Rotterdam
Hanover
Berlin
Poznań
Warsaw
Brussels
Cologne
Frankfurt am Main
Prague
Wrocław
Łódź
Lublin
Kraków
Luxembourg
Paris
Nantes
Loire
Bay of Biscay
Bordeaux
Toulouse
Bilbao
Lyon
Marseille
Zürich
Munich
Danube
Vienna
Bratislava
Brno
Košice
Budapest
Debrecen
Cluj-Napoca
Milan
Ljubljana
Zagreb
Belgrade
Timișoara
Craiova
Split
Niš
Sofia
Pristina
Podgorica
Musala
LATVIA
Daugavpils
LITHUANIA
Vilnius
Kaunas
Kaliningrad
Minsk
Grodno
Białystok
Bydgoszcz
Brest
Lviv
ROMANIA
SERBIA
MONTENEGRO
NORTH

Summary

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- largely untested

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- about an oversimplified algorithm

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Our existing theories are

- largely untested
- about an oversimplified algorithm
- ripe for improvement

Postdoc opening pre-announcement

- 1 position
- up to 24 months
- in Clermont-Ferrand, France
- no teaching required
- generous travel money
- starting date flexible

keep an eye on <u>dmanet</u>