IPCO 2024 summer school

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as a toy model for the simplex method we ask: for $a_1, \dots, a_n \in \mathbb{R}^2$ random, what is the expected number of vertices of the polygon

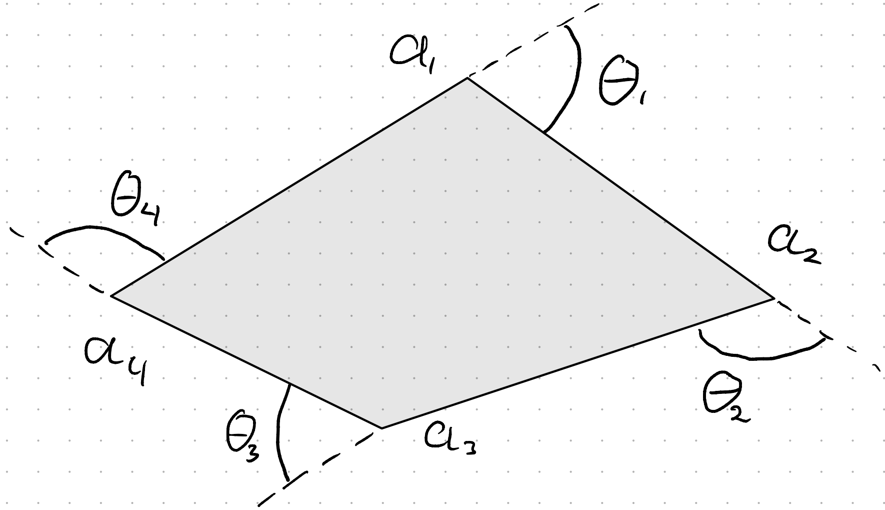
$$P = \operatorname{conv}(a_1, \dots, a_n).$$

1a. define $\theta_1, \dots, \theta_n$ as follows: if a_i is a vertex of P, then θ_i is the exterior angle at a_i.

if not then $\theta_i = 0$.

prove that

$$\mathbb{E}[\text{number of vertices of } P] \le \frac{2\pi}{\min_{i=1,\dots,n} \mathbb{E}[\theta_i \mid \theta_i > 0]}$$



1b. define $l_1, \dots, l_n$ as follows: if a_i is a vertex of P, then l_i is the sum length of the two edges of P incident to a_i.

if not then $l_i = 0$.

prove that

$$\mathbb{E}[\text{number of vertices of } P] \le \frac{4\pi\, \mathbb{E}[\max_{i=1,\dots,n} \|a_i\|]}{\min_{i=1,\dots,n} \mathbb{E}[l_i \mid l_i > 0]}$$

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1c. for $i=1,\dots,n$, define

$$d_i = \mathrm{dist}(a_i, \mathrm{conv}(a_1,\dots,a_{i-1}, a_{i+1},\dots,a_n))$$

note that $d_i > 0$ implies that a_i is a vertex of P.

assume $D>0$ is such that

$$\Pr[\, d_i \geq D \mid d_i > 0 \,] \geq 2/3$$

for all $i=1,\dots,n$.

prove that

$$\mathbb{E}[\text{number of vertices of } P] \leq O\left(\sqrt{\frac{\mathbb{E}[\max_{i=1,\dots,n} \|a_i\|]}{D}}\right).$$

consider the following classification of theories:

true and useful	false but useful
true but useless	false and useless

2a. write down one statement for every category.

2b. is this classification exhaustive (does every statement fit in exactly one category?)

is the classification useful?

discuss with your friends and neighbours